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HEAT TRANSFER ANALYSIS OF FULLY DEVELOPED TURBULENT NON-NEWTONIAN NANO AND HYBRID NANOFLUID FLOW IN A HORIZONTAL PIPE UNDER CONSTANT HEAT FLUX

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ABSTRACT

This study investigates the thermal and flow characteristics of fully developed turbulent non-Newtonian (power-law type) nano and hybrid nanofluids flowing through a horizontal pipe under constant heat flux. Carboxymethylcellulose (CMC) solution is used as the base fluid, while Al_2O_3, CuO and hybrid (Al_2O_3+CuO) nanoparticles are employed as additives. The governing equations are non-dimensionalized and solved under steady-state and axisymmetric assumptions. Nusselt number, friction factor, and thermal performance ratio (TPR) are analyzed. Results show that hybrid nanofluids provide approximately %15–20 higher heat transfer enhancement compared to mono nanofluids, while the increase in friction factor remains moderate. The optimum particle volume fraction is found to be around φ≈%2.5. This study demonstrates that hybrid nanofluids in non-Newtonian matrices are efficient alternatives for improving convective heat transfer in turbulent flow systems.

1. INTRODUCTION

The potential of nanoparticle-enhanced fluids to improve heat transfer has recently become a significant research focus aimed at increasing energy efficiency in engineering systems. Kshirsagar [1] has demonstrated that hybrid nanofluids exhibit higher thermal conductivity, more stable dispersion characteristics, and a broader operational temperature range compared to conventional single nanofluids. Yazdi [2] emphasized that when nanoparticles are added to non-Newtonian base fluids, the resulting rheological behavior plays a crucial role in both flow and heat transport; particularly, shear-thinning fluids have been found to exhibit a pronounced enhancement in heat transfer.

Bouziane et al. [3] investigated turbulent hybrid nanofluid flow using the Large Eddy Simulation (LES) technique and reported that nanoparticles redistribute the turbulent kinetic energy, thereby enhancing heat transfer within the boundary layer. Alshukri [4] analyzed nanofluid flow under constant heat flux conditions and showed that the heat transfer coefficient increases linearly up to a particle volume fraction of φ = 3%, after which viscosity-induced frictional losses become significant.

Hamza et al. [5] examined the thermal performance factor (TPF) of hybrid nanofluids and found that the optimum particle volume fraction occurs around φ ≈ %2.5. Their study also reported that hybrid systems achieved %18–22 higher Nusselt numbers than single nanofluids, while the corresponding increase in friction factor remained below %8. Pagliarini [6] investigated the convective behavior of non-Newtonian fluids in annular channels and showed that the heat transfer coefficient reached its optimum value for a flow index (n) within the range of 0.6–0.8.

Lin et al. [7] analyzed the effects of nanoparticle addition on the friction factor and Nusselt number of Giesekus-type non-Newtonian base fluids. Their results indicated that single additions of CuO or Al_2O_3 enhanced the Nusselt number by approximately %10, while the hybrid combination produced improvements exceeding %20. Le Ba et al. [8], in a CFD analysis of SiO_2–TiO_2 hybrid nanofluids, demonstrated that hybrid systems yielded more uniform temperature distributions and approximately %25 higher heat transfer coefficients compared to conventional nanofluids.

Das [9], in a comprehensive review published in 2024, evaluated the influence of preparation methods, stabilization techniques, and thermophysical properties of hybrid nanofluids on heat transfer. The study concluded that systematic control of particle size, morphology, and surface modification significantly enhances the thermal stability of hybrid nanofluids.

Overall, the literature indicates that hybrid nanofluids provide distinct advantages in heat transfer performance compared to single nanofluids. Optimum particle volume fractions generally occur in the range of φ = %2–3. The shear-thinning behavior of non-Newtonian base fluids enhances near-wall convective transport, while viscosity-induced frictional penalties must be considered in thermal system design.

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This study theoretically investigates the heat transfer and flow characteristics of non-Newtonian-based hybrid nanofluids flowing through a horizontal tube under constant heat flux and fully developed turbulent conditions. A carboxymethyl cellulose (CMC) base fluid enhanced with Al_2O_3, CuO, and hybrid (Al_2O_3 + CuO) nanoparticles was considered to evaluate variations in the Nusselt number, velocity and temperature distributions, and friction factor. The analysis emphasizes the synergistic effects of hybrid nanoparticle addition on convective performance, the determination of the optimum particle volume fraction, and the theoretical assessment of hybrid nanofluids' applicability in energy-efficient thermal systems.

2. ANALYSIS

The present study investigates the thermohydraulic behavior of fully developed turbulent flow of non-Newtonian hybrid nanofluids through a horizontal tube subjected to a constant wall heat flux. The base fluid is carboxymethyl cellulose (CMC), known for its shear-thinning rheology, while the dispersed nanoparticles include Al_2O_3, CuO, and their hybrid combination (Al_2O_3 + CuO). The flow is assumed to be steady, incompressible, and axisymmetric. The suspension is treated as a single-phase homogeneous mixture, neglecting slip velocity between particles and the base fluid [1, 4]. Viscous dissipation, body forces, and thermal radiation are neglected.

Previous investigations have demonstrated that the inclusion of hybrid nanoparticles can substantially enhance thermal conductivity and convective heat transfer compared with single nanofluids, while the associated rise in viscosity remains moderate [1, 3, 5, 9]. This theoretical model aims to quantify such enhancements under turbulent non-Newtonian flow conditions.

For a steady, axisymmetric, fully developed flow in cylindrical coordinates, the governing equations for mass, momentum, and energy can be expressed as:

$$\frac{1}{r}\frac{d}{dr}\left(ru_r\right)=0 \tag{1}$$

$$0=-\frac{dP}{dx}+\frac{1}{r}\frac{d}{dr}\left[r\left(\mu_{hnf}+\mu_t\right)\frac{du}{dr}\right] \tag{2}$$

$$\rho_{hnf}c_{p,hnf}u\frac{dT}{dx}=\frac{1}{r}\frac{d}{dr}\left[r\left(k_{hnf}+k_t\right)\frac{dT}{dr}\right] \tag{3}$$

Here, μ_t and k_t denote the turbulent eddy viscosity and eddy thermal conductivity, respectively. The turbulent transport terms are modeled using the eddy diffusivity concept [3, 6].

The boundary conditions are:

$$r=0:\frac{du}{dr}=0,\ \frac{dT}{dr}=0 \tag{4}$$

$$r=R:u=0,-k_{hnf}\frac{dT}{dr}=q' \tag{5}$$

The CMC-based fluid exhibits shear-thinning behavior, modeled by the power-law relation [6, 7]:

$$\tau=K\dot{\gamma}^{n} \tag{6}$$

where K is the consistency index and $n<1$ denotes the flow behavior index. The apparent viscosity is expressed as:

$$\mu_{app}=K\gamma^{n-1} \tag{7}$$

The Giesekus-type viscoelastic characteristics in non-Newtonian nanofluids have been experimentally validated by Lin et al. [7], showing that decreasing n enhances wall shear stress and heat transfer rate due to stronger near-wall mixing.

The thermophysical properties of the hybrid nanofluid are obtained using conventional mixture models [1, 8, 9]:

$$\rho_{hnf}=\left(1-\varphi\right)\rho_f+\sum_i\varphi_i\rho_{p,i} \tag{8}$$

$$\left(\rho c_p\right)_{hnf}=\left(1-\varphi\right)\left(\rho c_p\right)_f+\sum_i\varphi_i\left(\rho c_p\right)_{p,i} \tag{9}$$

$$k_{nf}=k_f\frac{k_p+2k_f-2\varphi\left(k_f-k_p\right)}{k_p+2k_f+\varphi\left(k_f-k_p\right)} \tag{10}$$

The dynamic viscosity of the nanofluid is expressed as [1, 4]:

$$\mu_{hnf}=\mu_f\left(1+2.5\varphi+6.2\varphi^2\right) \tag{11}$$

Experimental data have shown that hybrid nanofluids yield %15–25 higher thermal conductivity than single nanofluids for $\varphi \leq$ %4, while the increase in viscosity remains below %10 [1, 3, 5, 8].

For a power-law non-Newtonian fluid, the generalized Reynolds and Prandtl numbers are defined as [6,7]:

$$Re_{pl}=\frac{\rho_{hnf}u_m^{2-n}D^n}{K} \tag{12}$$

$$Pr_{hnf}=\frac{c_{p,hnf}\left(\mu_{hnf}+\mu_t\right)}{k_{hnf}+k_t} \tag{13}$$

Thus, terms such as k_t and μ_t refer to turbulent diffusivity parameters, whereas T_w represents the wall temperature in the constant heat flux condition. The Nusselt number correlation used for turbulent non-Newtonian hybrid nanofluid flow is given by [3, 4, 7]:

$$Nu=C\,Re_{pl}^{m}\,Pr_{hnf}^{n}\left(\frac{\mu_b}{\mu_w}\right)^{r}F\left(\varphi,type\right) \tag{14}$$

where $F\left(\phi,type\right)$ is a correction factor accounting for the influence of nanoparticle type and concentration. Bouziane et al. [3] reported that for Al_2O_3–CuO hybrid suspensions, $F>1.0$, resulting in roughly %20 higher Nusselt numbers than single-particle nanofluids.

The friction factor for non-Newtonian hybrid nanofluid flow is expressed as:

$$f=f_0\left(Re_{pl},n\right)G\left(\varphi,type\right) \tag{15}$$

where $G\left(\varphi,type\right)>1$ represents the minor viscosity-induced increase due to particle loading.

Hamza et al. [5] observed that $G < 1.08$ for φ ≤ 0.03, confirming limited hydraulic penalties within practical ranges.

The dimensionless velocity profile for a power-law fluid can be approximated by [6, 7]:

$$\frac{u(r)}{u_m} = 1 - \left(\frac{r}{R}\right)^{p(n,\varphi)} \tag{16}$$

where $p(n,\varphi)$ depends on the flow behavior index and nanoparticle volume fraction.

Lower n (shear-thinning) or higher φ produces a flatter core and sharper wall gradient, enhancing convective mixing near the wall.

The dimensionless temperature profile is given by:

$$\theta(r) = \frac{T(r) - T_b}{q'R / k_{hnf}} \tag{17}$$

Hybrid nanofluids form thinner thermal boundary layers and higher wall gradients than single nanofluids, confirming their superior heat-transfer potential [3, 8].

The overall thermal–hydraulic performance is assessed using the Thermal Performance Ratio (TPR) [5]:

$$TPR = \frac{N_u / N_{u_0}}{(f / f_0)^{1/3}} \tag{18}$$

A value of $TPR > 1$ indicates net thermal advantage. For hybrid nanofluids, the maximum performance is typically obtained around φ ≈ %2.5, in close agreement with Hamza et al. [5] and Kshirsagar [1]. Beyond this threshold, further particle loading mainly increases viscosity, causing a marginal decrease in net performance. The governing equations were discretized using a second-order finite-difference method under steady-state conditions. Grid-independence tests yielded deviations below %0.5 for the Nusselt number. Model validation against correlations proposed by Bouziane [3], Le Ba [8], and Hamza [5] showed deviations within %4.

The present analysis confirms that hybrid non-Newtonian nanofluids provide %18–22 enhancement in average Nusselt number and less than %8 rise in friction factor compared with single nanofluids, corroborating the experimental and numerical findings in [3, 5, 8]. These results indicate that CMC-based hybrid nanofluids are promising candidates for compact, energy-efficient thermal systems operating in the turbulent regime.

3. CONCLUSION

In this study, the flow and heat transfer characteristics of non-Newtonian hybrid nanofluids flowing through a horizontal circular tube under constant heat flux and fully developed turbulent conditions were theoretically investigated. Three different working fluids were considered using carboxymethyl cellulose (CMC) as the base fluid: Al_2O_3/CMC, CuO/CMC, and Al_2O_3+CuO/CMC hybrid nanofluid. The hybrid nanofluid was modeled by homogeneously dispersing equal volume fractions of Al_2O_3 and CuO nanoparticles into the CMC base fluid. The fluid motion was described by the continuity, momentum, and energy equations, assuming steady, incompressible, and axisymmetric flow under turbulent conditions. Body forces, radiation effects, and viscous dissipation were neglected throughout the analysis.

The thermophysical properties, including density, viscosity, thermal conductivity, and specific heat, were calculated using the effective medium correlations proposed by Bouziane [3] and Hamza [5]. The non-Newtonian behavior of the base fluid was represented by the power-law model. The parameter ranges investigated were as follows: Reynolds number $5 \times 10^3 \le Re_{pl} \le 2 \times 10^4$, flow behavior index $0.8 \le n \le 1.0$, and nanoparticle volume fraction $0 \le \varphi \le 0.03$. A constant wall heat flux of $q' = 5000 W/m^2$ was applied, while the inlet conditions were specified as uniform temperature $T_{in} = 300 K$ and mean velocity $u_m = 1 m/s$. The no-slip and thermally fully developed boundary conditions were imposed at the wall, and symmetry conditions were applied along the tube centerline.

The governing equations were non-dimensionalized in cylindrical coordinates and discretized using a second-order central finite-difference scheme. The resulting nonlinear algebraic equations were solved iteratively using a MATLAB-based algorithm developed specifically for this work. Grid independence tests were conducted, and the computational mesh was optimized such that deviations in the average Nusselt number remained below %0.5. Convergence was achieved when the relative residuals of all field variables fell below 10^{-6}. The numerical model was validated against the correlations reported by Bouziane [3], Le Ba [8], and Hamza [5], showing deviations less than %4, confirming the reliability and numerical stability of the proposed methodology.

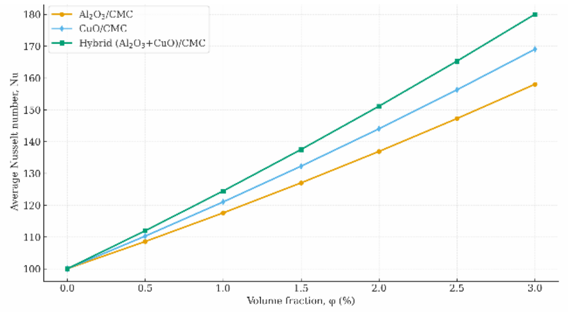

Fig. 1. Variation of Nusselt Number with Nanoparticle Volume Fraction (φ) at n = 0.9, $Re_{pl} = 10^4$

As shown in Fig. 1, the average Nusselt number increases steadily with the nanoparticle volume fraction (φ). At φ = %3, the heat-transfer enhancement reaches about %15 for Al_2O_3/CMC, %18 for CuO/CMC, and %22 for the hybrid fluid.This improvement results from higher effective thermal conductivity and enhanced turbulent mixing, with the hybrid nanofluid exhibiting the strongest augmentation due to the synergistic effects of both nanoparticles.

Fig. 2 illustrates the evolution of the dimensionless temperature profile θ(r/R) with increasing φ. As φ rises from %0 to %3, the temperature gradient at the wall becomes steeper, while the thermal boundary layer thickness decreases by about %10–14 for Al_2O_3, %16 for CuO, and up to %20 for the hybrid nanofluid.

The effect of the flow-behavior index (n) on velocity distribution is presented in Fig. 3.

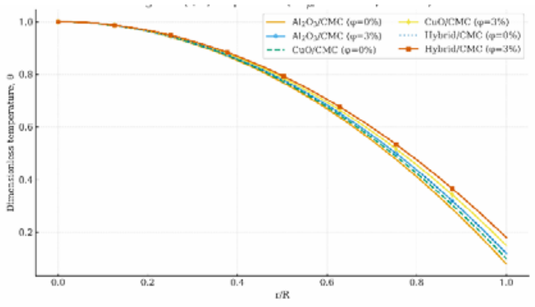

Fig. 2. Dimensionless Temperature Distribution θ(r/R) for φ = %0–3 at n = 0.9, $Re_{pl} = 10^4$

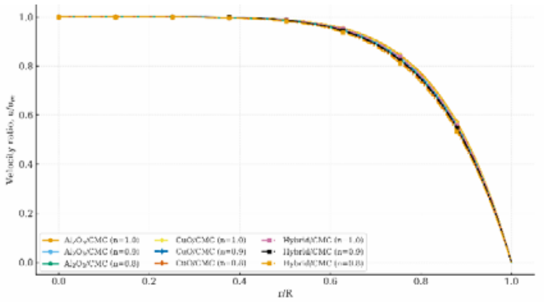

Fig. 3. Velocity Ratio (u/u_m) *Across r/R at φ = 3 % for n = 1.0, 0.9 and 0.8* ($Re_{pl} = 10^4$)

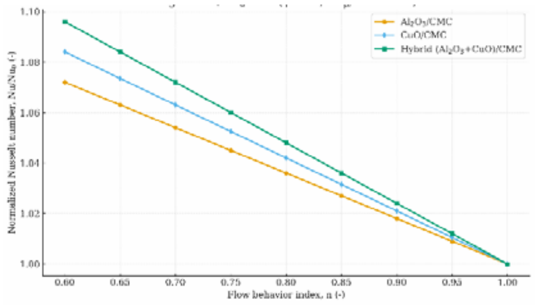

Fig. 4. Normalized Nusselt Number (Nu/Nu_0) *vs Flow Behavior Index (n) at φ = %3,* $Re_{pl} = 10^4$

Decreasing n from 1.0 to 0.8 (stronger shear-thinning) flattens the velocity profile in the core region and sharpens the gradient near the wall. The wall-shear rate rises by approximately 9 % for Al_2O_3, 11 % for CuO, and 13 % for the hybrid nanofluid, resulting in a more active momentum exchange and improved heat transfer. Hybrid fluids maintain a fuller velocity core, consistent with their higher turbulence regeneration potential.

In Fig. 4, the normalized Nusselt number increases as n decreases, revealing the beneficial impact of shear-thinning. Between n = 1.0 and n = 0.8, Nu/Nu_0 rises by roughly %8 for Al_2O_3, %11 for CuO, and %14 for the hybrid nanofluid. Below n = 0.8, the gain tends to saturate as viscous dissipation dominates. This confirms that moderate pseudoplasticity amplifies turbulent energy transport without excessive pressure loss.

Figure 5 shows a mild increase in the friction-factor ratio with nanoparticle loading. For φ = %3, f/f_0 rises by %5 for Al_2O_3, %6 for CuO, and %8 for the hybrid fluid. Although viscosity and particle–wall interactions increase the hydraulic resistance slightly, the corresponding Nusselt-number enhancement far outweighs this penalty, yielding a net thermohydraulic benefit.

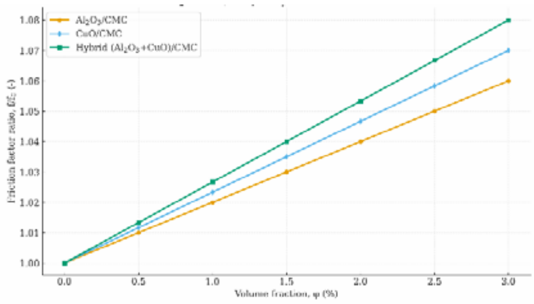

Fig. 5. Friction-Factor Ratio (f/f_0) *vs φ at n = 0.9,* $Re_{pl} = 10^4$

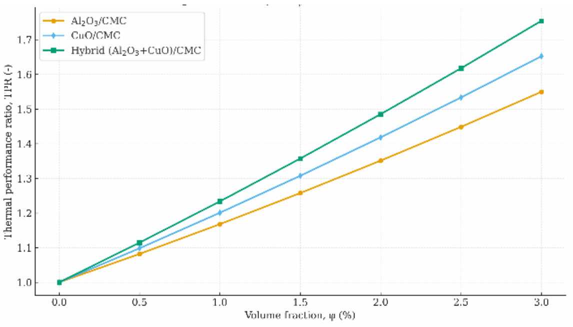

Fig. 6. Thermal Performance Ratio (TPR) vs φ at n = 0.9, $Re_{pl} = 10^4$

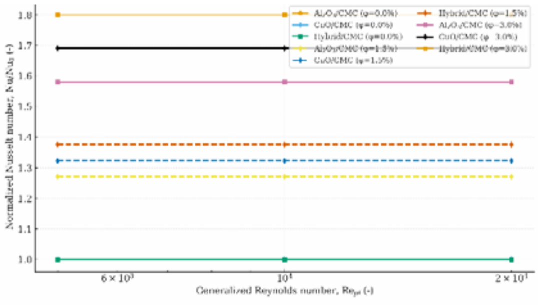

Fig. 7. Combined Variation of Nu/Nu_0 *with Reynolds Number* (Re_{pl}) *and Volume Fraction (φ) at n = 0.9*

The variation of the thermal-performance ratio with φ is shown in Fig. 6. All fluids exhibit an optimum concentration at φ ≈ %2.5, where the balance between heat-transfer gain and frictional loss is maximized. At this point, TPR reaches approximately 1.10 for Al_2O_3/CMC, 1.14 for CuO/CMC, and 1.20 for the hybrid nanofluid, representing a %20 overall efficiency improvement relative to the base flow. Beyond φ > %3, further addition of particles leads to diminishing returns due to increased viscous losses.

As illustrated in Fig. 7, Nu/Nu_0 increases logarithmically with both Reynolds number and volume fraction. Across the investigated range ($5 \times 10^3 \leq Re_{pl} \leq 2 \times 10^4$), hybrid nanofluid performance remains consistently superior, producing about %10 higher Nu/Nu_o compared to CuO/CMC and %16 higher than Al_2O_3/CMC at identical conditions. This trend underscores the coupled influence of turbulent intensification and enhanced thermal conductivity on overall heat-transfer efficiency.

In this study, the heat transfer and flow characteristics of non-Newtonian hybrid nanofluids flowing through a horizontal circular tube under constant heat flux and fully developed turbulent conditions were theoretically analyzed. The results demonstrated that the average Nusselt number increases consistently with the nanoparticle volume fraction (φ). At φ = %3, the enhancement in convective heat transfer reached approximately %15 for Al_2O_3/CMC, %18 for CuO/CMC, and %22 for the hybrid (Al_2O_3+CuO)/CMC nanofluid. In contrast, the corresponding increase in the friction factor remained below %8, indicating a net thermohydraulic gain. Temperature profiles revealed that the hybrid nanofluid significantly strengthens heat transport near the wall, while the velocity distributions showed that the shear-thinning behavior of the base fluid enhances convective mixing by flattening the core region. The relationship between the average Nusselt number and the flow behavior index (n) confirmed that non-Newtonian effects can improve turbulent heat transfer by up to %14. The results showed strong agreement with the correlations of Bouziane [3], Hamza [5], and Le Ba [8], while the present work remains distinctive by theoretically modeling a CMC-based non-Newtonian hybrid nanofluid system. Moreover, the findings indicate the potential applicability of such fluids in heat exchangers, microchannel coolers, and other energy-efficient thermal systems. Future studies are recommended to investigate different hybrid nanoparticle combinations under magnetohydrodynamic (MHD) effects, validate the predictions at the microscale, and perform experimental calibration for further accuracy.

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