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# A COMPREHENSIVE OVERVIEW OF THE MECHANICS AND APPLICATIONS OF DOUBLE-BEAM SYSTEMS

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ABSTRACT

*Double-beam systems have attracted significant attention due to their unique mechanical behavior and wide range of engineering applications. This study offers an extensive review of the mechanics and applications of double-beam systems, addressing both theoretical progress and practical uses. Initially, the theoretical foundations, including Euler-Bernoulli and Timoshenko beam theories, higher-order shear deformation models, and advanced non-local elasticity and strain gradient theories for micro- and nano-scale applications are reviewed. In addition, the role of elastic and viscoelastic interlayers-modeled by Kelvin-Voigt, Maxwell, Burgers, and Zener approaches is discussed, as well as the influence of different foundation models such as Winkler, Pasternak, and Kerr. The literature survey traces the historical evolution of double-beam research, from early vibration and stability studies in the 1970s to recent advances involving functionally graded materials, cracked beam analyses, and moving load problems. Future research directions include the development of smart material-based double-beam systems, broader adoption of computational methods, and coupling theoretical models with experimental approaches. These perspectives underline the importance of double-beam systems not only in structural mechanics but also in advanced engineering applications.*



## 1. INTRODUCTION

Double-beam systems are structural configurations formed by connecting two parallel beams through an elastic or viscoelastic interlayer. Although single-beam models can be applied to many engineering problems, they are often insufficient to accurately reflect the real behavior, particularly in multilayered structures or under complex loading conditions. Therefore, double-beam systems have emerged as an important field of research in modern engineering, both from theoretical and practical perspectives. The investigation of double-beam systems is critical not only in terms of mechanical strength but also for understanding the dynamic behaviors such as vibration, buckling, stability, and wave propagation. The manner in which the two beams are connected and the properties of the interlayer material directly determine the mechanical performance of the system. In particular, viscoelastic interlayers are increasingly preferred in engineering applications due to their vibration-damping capacity.

The significance of these systems is further emphasized by their wide range of applications across various engineering disciplines. In civil engineering, they are used in bridge decks, multilayer floor systems, and beams resting on elastic foundations; in mechanical engineering, they are applied in the design of composite panels and vibration control elements; in biomechanics, they are utilized for modeling bone-implant systems; and in nano-scale engineering.

In recent years, with the advances in computer-aided analysis and the development of the finite element method (FEM), research on double-beam systems has become more comprehensive. Different foundation models, variable interlayer materials, functionally graded structures (FGMs), and nano-scale applications have been contributed. In this context, double-beam systems are not only essential for improving current engineering solutions but also hold great potential for the design of smart materials and structures of the future.

The double-beam system considered in this study consists of two parallel beams connected by an elastic interlayer, as shown in Fig. 1 [1].

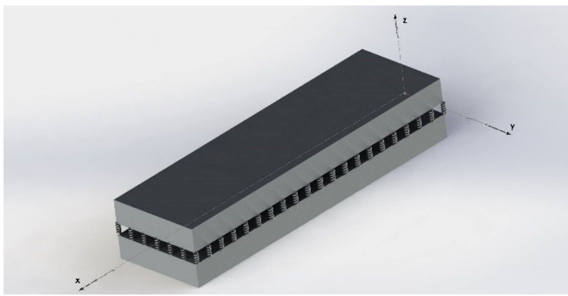

*Fig. 1. 3-D representation of the double-beam system in the global coordinate system*

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## 2. THEORETICAL FRAMEWORK

Double-beam systems are modeled as two parallel beams connected by an elastic or viscoelastic interlayer. Compared to single-beam models, the configuration exhibits more complex behavior and enables a more realistic representation of multilayered structures. The analysis of double-beams is of critical importance not only under static loads but also in dynamic problems such as vibration, buckling, wave propagation, and stability. From the perspective of beam theories, the earliest approaches are based on the Euler-Bernoulli theory. While the theory provides adequate results for slender beams with small deflections, it neglects shear deformations and rotary inertia, making it insufficient for moderately thick beams. This limitation was largely addressed by the Timoshenko theory, which incorporates shear deformation and rotary effects into the model. More recently, higher-order shear deformation theories have been developed to achieve more accurate results. The schematic representation of the double-beam system is illustrated in Figure 2. The configuration consists of two parallel beams, referred to as Beam 1 and Beam 2, interconnected by an elastic or viscoelastic layer under clamped and free boundary conditions. The system is subjected to a harmonic excitation force ( $F_0 e^{i\omega t}$ ) acting on the upper beam, inducing transverse wave propagation along the longitudinal ( $x-$ ) direction. The configuration serves as a representative model for analyzing vibration, wave propagation, and dynamic coupling phenomena in double-beam systems.

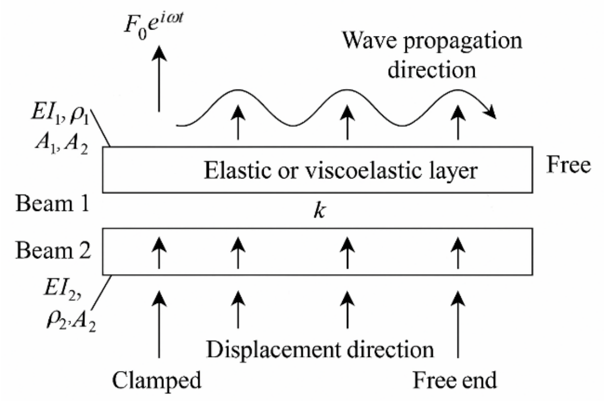


*Fig. 2. Schematic representation of a doub2le-beam system coupled through an elastic or viscoelastic interlayer under harmonic excitation*

In the mathematical modeling of double-beam systems, the principal methodology involves deriving the governing equations of motion for each individual beam and subsequently coupling them through the influence of the interlayer. In the general form, the governing equations of motion for each beam can be expressed as follows:

$$EI_1 \frac{\partial^4 w_1}{\partial x^4} + \rho_1 A_1 \frac{\partial^2 w_1}{\partial t^2} + K(w_1 - w_2) = 0 \quad (1)$$

$$EI_2 \frac{\partial^4 w_2}{\partial x^4} + \rho_2 A_2 \frac{\partial^2 w_2}{\partial t^2} + K(w_2 - w_1) = 0 \quad (2)$$

In Equations (1-2), " $EI_i$ " ( $i = 1,2$ ) represents the bending rigidity, " $\rho_i$ " denotes the mass density, " $A_i$ " the cross-sectional area, " $w_i(x,t)$" the transverse displacements, and " $K$ " the coupling stiffness of the connecting layer of each beam. The parameters highlight that the vibration behavior of the beams is governed not only by their individual rigidity and mass parameters but also by the mutual interaction through the interlayer. In the dynamic analysis of double-beam systems, wave propagation constitutes a fundamental aspect. When the beam displacements are represented in a harmonic wave form as $w_i(x,t) = W_i e^{i(kx-\omega t)}$, the dispersion relation of the system is derived as follows:

$$(EI_1 k^4 - \rho_1 A_1 \omega^2 + K).(EI_2 k^4 - \rho_2 A_2 \omega^2 + K) - K^2 = 0 \quad (3)$$

In Equation (3), the dispersive nature of wave propagation and the variation of frequency with respect to wave number are explained. " $k$ " denotes the wave number and " $\omega$ " represents the vibration frequencies, respectively. Namely, the identification of resonance regions and the analysis of wave transmission characteristics are carried out.

## 3. LITERATURE REVIEW

The earliest comprehensive studies on double-beam systems primarily focused on free vibration analysis under various boundary conditions. Kim et al. [2] investigated functionally graded double-beam systems (FGDBS) connected by an elastic layer and validated their method through comparisons with FEM and previous literature. Zhang and Shi [3] presented an analytical model of laminated composite double-beam systems (LCDBS) and examined their free vibration behavior using an exact solution based on an improved Fourier series method. Kozić et al. [4] developed an analytical theory employing a Kerr-type three-parameter model to describe the dynamic characteristics of axially compressed elastically connected beams. Oniszczuk [5] analyzed the forced transverse vibrations of elastically connected double-plate systems, enabling the determination of resonance and vibration absorption conditions. Zhao and Chang [6] provided a systematic closed-form solution for free and forced vibrations of general double-beam systems with arbitrary boundary conditions. Mao [7] extended the analysis to multiple parallel Euler-Bernoulli beams joined by a Winkler-type elastic layer, applying the Adomian Modified Decomposition Method (AMDM). Hamada et al. [8] analyzed the free and forced vibrations of a system consisting of two elastically connected parallel upper and lower beams with unequal masses and unequal flexural rigidities, by using the generalized finite integral transformation and Laplace transformation methods. The elastically connected multi-beam system was compared with a multi-degree-of-freedom mass-spring system, and the similarity between the two models was demonstrated.

Considerable research has also been devoted to the influence of viscoelastic interlayers on the dynamic behavior of double-beam systems. Li et al. [9] proposed a general viscoelastic interlayer capable of capturing complex stiffness and damping effects, and introduced a novel state-space approach with a mode-shape constant. Mao and Wattanasakulpong [10] investigated the free vibration and stability of clamped double-beam systems joined by a

Winkler-type elastic layer using AMDM. Han et al. [11] analyzed the damping characteristics of viscoelastic double-beam systems depending on the dynamic stiffness method combined with the Wittrick-Williams algorithm. Li et al. [12] presented a semi-analytical method to investigate the natural frequencies and mode shapes of a double-beam system interconnected by a viscoelastic layer. To verify the efficiency of the proposed approach, several numerical examples were provided and discussed in detail. This method contributes to characterizing the dynamic responses of double-beam structures and supports further design and optimization studies. Oniszczuk [13] theoretically studied damped transverse vibrations of a double-string system connected through a Kelvin-Voigt type viscoelastic layer.

Research has also explored cracked systems and stability phenomena. Chen et al. [14] derived closed-form solutions for steady-state forced vibrations of cracked double-beam systems resting on a Winkler-Pasternak foundation, highlighting the effects of crack geometry and interlayer stiffness. Rahmani et al. [15] investigated the buckling behavior of bonded functionally graded nanobeams under compressive axial loads using Eringen's nonlocal elasticity theory and the Euler-Bernoulli model. Mirzabeigy [16] derived an analytical formula to estimate the natural frequencies of a simply supported double-beam system containing an open crack. The Euler-Bernoulli beam hypothesis was applied to the beams, while the Winkler model was used for the intermediate layer. By employing the admissible shape functions and the Rayleigh method, an explicit expression for the natural frequencies was obtained. Furthermore, to verify the accuracy of the analytical formulation, the problem was re-solved using the Differential Transform Method (DTM).

With the advancement of nonlocal and scale-dependent theories, nano- and micro-scale double-beam systems have attracted significant attention. Ahmadi [17] studied the free vibrations of multiple nanobeam systems under different boundary conditions and examined the effects of parameters such as coupling stiffness, nonlocal parameters, and the number of nanobeams. Lu et al. [18] developed a size-dependent sinusoidal shear deformation beam model based on the nonlocal strain gradient theory to investigate the free vibration of nanobeams, comparing results with classical and non-classical beam theories.

Finally, studies have addressed parametric resonance and dynamic stability in double-beam systems. Chen et al. [19] examined the transverse parametric vibrations of an axially accelerating viscoelastic beam under tension. Stability conditions were derived and confirmed through numerical simulations, with parametric studies illustrating the influence of dynamic viscosity, mean axial speed, and axial tension. Asiri [20] performed dynamic, fatigue, and harmonic analyses for different combinations of beam cross-section geometries in a two-beam contact system. A computer-aided finite element software was employed to numerically model the stresses and deformations developed within the double-beam configuration. Three combinations of beam cross-sections such as square-square, circular-circular, and square-circular were simulated. The results provided valuable insights for design engineers in determining the optimal combination of beam cross-sections that ensures superior performance in structural and machine design applications involving double-beam systems. Rangel et al. [21] presented a new design and performance evaluation of an energy harvester. The generator was designed as a double-beam mechanical structure on which eight piezoelectric elements were attached and subjected to cyclic tensile and compressive loads. Before the practical manufacturing of the device, the geometric dimensions of the beam structure were optimized using finite element analysis. Numerical and experimental results regarding the dynamic behavior of the generator and the generated electric voltage were presented and compared.

## 4. CONCLUSIONS

Research on double-beam systems has mainly concentrated on beam theories, viscoelastic models (Kelvin-Voigt, Maxwell, Burgers, Zener), elastic foundation models (Winkler, Pasternak), as well as nano/micro-scale theories and modern approaches such as artificial intelligence and optimization methods. Between 1970 and 2000, theoretical models and the first numerical solutions were developed, paving the way for later applications of viscoelastic layers, advanced foundation models, functionally graded materials (FGMs), and nano-scale theories. From 2000 to 2010, numerical methods such as the finite element method (FEM) and differential transformation method (DTM) became more prominent, providing reliable solutions under complex boundary conditions. The period 2010-2020 saw diversification with nano-scale approaches, FGMs, cracked systems, moving loads, and advanced viscoelastic models, although studies remained largely theoretical with limited experimental validation. Since 2020, research has shifted toward modern engineering applications, with computational methods, nano/micro-scale theories, and energy harvesting, which indicate promising directions but are still in early stages with limited practical integration.

The conducted review and analysis on double-beam systems lead to the following key conclusions:

- **Evolution of Research Scope:** Early investigations were primarily based on classical beam theories. Further studies introduced viscoelastic interlayers, elastic foundations, functionally graded materials (FGMs), and nano-scale theories, significantly broadening the scope of double-beam system research.
- **Engineering Significance:** Double-beam systems are not merely theoretical constructs but practical models applied to multilayer structures, bridge decks, composite panels, energy systems, and nano devices.
- **Analytical Advantages:** Compared with single-beam models, double-beam formulations offer a more accurate representation of vibration, buckling, stability, and energy dissipation behaviors, enabling improved structural analysis.
- **Current Limitations:** Most existing studies are limited to idealized boundary conditions, homogeneous material assumptions, and purely theoretical or numerical approaches. Experimental validations and the analysis of complex damage scenarios remain insufficient.
- **Emerging Trends:** Recent research has incorporated artificial intelligence, optimization techniques, piezoelectric energy harvesting, and nano-scale modeling,

presenting new opportunities to overcome traditional modeling limitations.

- **Future Outlook:** Double-beam systems are expected to play a crucial role in the development of smart, adaptive, and sustainable structures, combining advanced materials and computational intelligence.

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