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STACKING SEQUENCE OPTIMIZATION WITH FAILURE CRITERIA

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ABSTRACT

Optimum stacking sequences of laminated composite beams are investigated for minimum normal and shear stresses by the use of two different failure criteria for different boundary conditions. The stacking sequence optimization is carried out by genetic algorithm (GA) optimization technique. Fiber orientation angles are used as the design parameters. During the optimization process, Tsai-Wu and Tsai-Hill failure criteria are both used as a control mechanism in the algorithm for each generation. Since the minimization of bending stress parameters and failure analysis are performed at the same time, convergence and reliability of the optimum stress values are improved respectively.

INTRODUCTION

Depending on the type and purpose of use, the structures may be subjected to various effects (static loading, dynamic loading, impact etc.) and as a result of these effects, different mechanical behaviors may also take place. In engineering structures, it is desirable that the material can give the most appropriate response or behavior in order to prevent failure from any external effects. In other words, it is aimed to obtain the design parameters that can be run at a determined life before the structure is damaged. Since the fiber reinforced composite materials have an anisotropic and heterogeneous structures, they should be analyzed within certain failure criteria. Mines and Alias [1] investigated the stress analysis of military vehicles in which outer layers are made of hardened glass epoxy material (SE84) and inner layers are made of foam materials H100 and H200. Hayes and Lesko [2] studied on a model to describe failure types of composite beams used in daily life and make it possible to predict strength parameters used in fatigue life prediction more accurately. Naik, et al [3] investigated the minimum weight designs of composite layers by use of maximum stress and Tsai-Wu failure criteria. With the new model developed, it was tried to increase the effectiveness of the failure criterion. Lopez et al [4] analyzed the optimal structure behavior using first layer failure criterion in the optimization of layered composite structures. They performed the minimum weight and cost optimization of layered composite structures by taking the maximum stress, Tsai-Wu and Puck failure criteria into consideration. Santiuste et al [5] compared Hou and Hashin failure criteria for composite beams damaged by impact at low speed through three-point bending test machine under dynamic conditions. Naik et al [6] attempted to optimize the laminated composite structures using optimization techniques such as vector set optimization and genetic algorithm. Optimization process was terminated by minimizing the weight and using different failure criteria.

The objective of this study is to perform the stacking sequence optimization of laminated composite beams by taking Tsai-Wu and Tsai-Hill failure criteria into consideration in order to obtain the minimum bending stresses.

ANALYTICAL METHOD

In the present study, three layered beams having rectangular cross-section, constant thickness of h, unit width, length of L and a transverse distributed load of q(x) on its top plane are considered. The coordinate system is placed in the mid-plane where $0 \leq x \leq L$ and $-h/2 \leq z \leq h/2$ as illustrated in Figure 1.

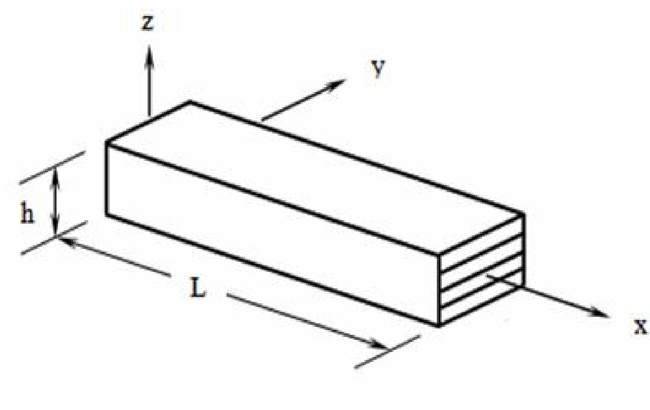

Fig. 1. Beam geometry and coordinate axes

The displacement fields are assumed to be in parallel with the general shear deformation shell theory [7] and given as follows:

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$$U(x,z) = u(x) - z\,w_{,x} + \emptyset(z)\,u_1(x)$$
$$W(x,z) = w(x) \tag{1}$$

U and W represent the displacement fields with respect to x and z. u, u_1 and w are the unknown displacement functions of the mid-plane. $_{,x}$ denotes the differentiation with respect to x. $\emptyset(z)$ is the shape function and chosen as a cubic function of layer thickness. The state of stresses in each k^{th} layer are given by generalized Hooke's Law and kinematic relations [8] as follows:

$$\begin{bmatrix}\sigma_x^{(k)}\\ \tau_{xz}^{(k)}\end{bmatrix} = \begin{bmatrix}Q_{11}^{(k)} & 0\\ 0 & Q_{55}^{(k)}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \gamma_{xz}\end{bmatrix} \tag{2}$$

Terms of "$Q_{ij}^{(k)}$" are the transformed reduced stiffnesses depending on materials properties such as elasticity modulus (E_{ij}), shear modulus (G_{ij}) and Poisson ratio (υ_{ij}) [9]. By use of stress-strain relations into force and moment definitions, the constitutive equations given in detail in Ref. 8 are obtained. The governing equations used in the study is given as below:

$$N_{x,x} = 0$$
$$M_{x,xx} = q(x) \tag{3}$$
$$M^a_{x,x} - Q^a_x = 0$$

By using the constitutive equations into governing equations and applying the boundary conditions which can be prescribed at the edges of the beam where x=0 and x=L for simply supported (S), cantilever (C) and free (F) cases simultaneously, three unknown displacement functions with eight integration constants are obtained as follows:

$$u_1(x) = C_1 e^{-px} + C_2 e^{px} - \frac{(qx + C_3)D_{11_1}}{A_{55}D_{11}}$$

$$u(x) = -\frac{B_{111}}{A_{11}}u_1(x) + C_7 x + C_8$$

$$w(x) = \frac{D_{111}}{pD_{11}}[-C_1 e^{-px} + C_2 e^{px}] - \frac{D_{11_1}}{pD_{11}\left[\frac{p}{A_{55}}\left(\frac{qx^2}{2} + C_3 x\right)\right]} + \frac{1}{D_{11}}\left(\frac{qx^4}{24} + C_3\frac{x^3}{6}\right) + C_4\frac{x^2}{2} + C_5 x + C_6 \tag{4}$$

$$p = \sqrt{\frac{-A_{55}A_{11}D_{11}}{D_{111}^2 A_{11} - D_{1111}A_{11}D_{11}}}$$

Tsai-Wu and Tsai-Hill failure criteria are commonly used in the stress analysis of layered composite beams. As long as the equilibrium expressed by normal and shear stresses of any point, maximum tensile, compressive and shear strengths are satisfied, failure does not occur in the structure. For a composite beam, Tsai-Wu

$$F_1\sigma_{xx} + F_{11}\sigma_{xx}^2 + \frac{\tau_{xz}^2}{S} \le 1$$
$$F_1 = \left(\frac{1}{X_T} - \frac{1}{X_C}\right), F_{11} = \left(\frac{1}{X_T X_C}\right) \tag{5}$$

and Tsai-Hill

$$\left(\frac{\sigma_{xx}}{X_T}\right)^2 + \left(\frac{\tau_{xz}}{S}\right)^2 \ge 1 \tag{6}$$

failure criteria can be simplified and given as below. The terms σ_{xx} and τ_{xz} are defined as the in-plane normal and shear stresses, F_1 and F_{11} as strength parameters, X_T and X_C as the maximum tensile and compressive strengths in x- direction and S as the shear strength respectively.

OPTIMIZATION METHOD

Genetic algorithm is an evolutionary optimization technique that is firstly used by Goldberg [10] and Hajela [11] in structural optimization. It generally depends on improving of a random initial population by genetic operators such as reproduction, crossover and mutation. In the study, fiber orientation angles (θ(k)) are chosen as the design parameters where k represents the number of layer. The optimization process is performed for two cases. In the first case, the minimum bending stresses and corresponding stacking sequences are obtained without taking failure criteria into consideration. A random initial population including the stacking sequences is generated and the individuals are ranked from minimum to maximum after each generation. Then genetic operators are applied until the targeted value is obtained for a specific number of generation. In the second case, the failure criteria are inserted into the algorithm, after each generation the minimum bending stresses of the corresponding stacking sequence are calculated and validity of the optimum values are controlled for Tsai-Wu and Tsai-Hill failure criteria.

NUMERICAL RESULTS

In the optimization problem, the composite beam is assumed to be constructed of graphite/epoxy layers with a length of L=1 m, thickness of h=0.02 m and under a uniform transverse distributed load of $q(x)$=1000 N/m. Mechanical properties are given as follows [12]:

$$E_{11}=241.5\text{ GPa},\ E_{22}=E_{33}=18.89\text{ GPa},$$
$$G_{23}=5.18\text{ GPa},\ G_{12}=G_{13}=3.45\text{ GPa},$$
$$X_T=241.5\text{ GPa},\ X_C=18.89\text{ GPa}, \tag{7}$$
$$S=18.89\text{ GPa}$$
$$\upsilon_{23}=0.25,\ \upsilon_{12}=\upsilon_{13}=0.24$$

In Table 1 and Table 2, the minimum normal and shear stresses and corresponding stacking sequences are presented for different boundary conditions. The optimization process is performed with a population of 50 individuals and a fiber increment of 10° at certain cross-sections of the beam. The normal stresses are obtained at x=L/2 and z=-h/2, shear stresses are obtained at z=-h/8, and x=L/4 for C – C, C – F and S – S boundary conditions respectively. In the first case, stress values are obtained without taking the failure criteria into consideration and most of the results are identical with the exact solution. In the second case, by use of the failure criteria, all of the stress values are obtained identically with the exact solution.

Table 1. *Minimum normal stresses (σ_{xx}) and corresponding stacking sequences for different boundary conditions*

Boundary Condition		σ_{xx} [Pa]	Stacking Sequence
C – C	**Exact Value**	1704.20	70°/50°/0°
	Min. Value (1)	41783.80	30°/50°/0°
	Min. Value (2)	1704.20	70°/50°/0°
C – F	**Exact Value**	260712.70	90°/90°/0°
	Min. Value (1)	260712.70	90°/90°/0°
	Min. Value (2)	260712.70	90°/90°/0°
S – S	**Exact Value**	147395.60	80°/80°/10°
	Min. Value (1)	147395.60	80°/80°/10°
	Min. Value (2)	147395.60	80°/80°/10°

Table 2. *Minimum shear stresses (τ_{xz}) and corresponding stacking sequences for different boundary conditions*

Boundary Condition		τ_{xz} [Pa]	Stacking Sequence
C – C	**Exact Value**	0.20	40°/10°/80°
	Min. Value (1)	0.20	40°/10°/80°
	Min. Value (2)	0.20	40°/10°/80°
C – F	**Exact Value**	18.60	10°/80°/70°
	Min. Value (1)	18.60	10°/80°/70°
	Min. Value (2)	18.60	10°/80°/70°
S – S	**Exact Value**	1170.10	30°/90°/70°
	Min. Value (1)	4709.40	10°/70°/50°
	Min. Value (2)	1170.10	30°/90°/70°

Optimization process is performed throughout 20 generations by minimizing the stress values in each generation. For a three layered beam and fiber increment of 10°, there are 1000 possible stacking sequences and optimum values are obtained among these. Since there will be too many possible stacking sequences for smaller fiber increment angles and many number of layers, efficient use of the algorithm will be inevitable. Thus, the efficiency of algorithm is increased by Tsai-Wu and Tsai-Hill failure criteria. As illustrated in the figures, the use of failure criteria generally leads to better results in earlier generations. The stacking sequences and corresponding normal and shear stresses are optimized for two cases and they are compared with the exact solution.

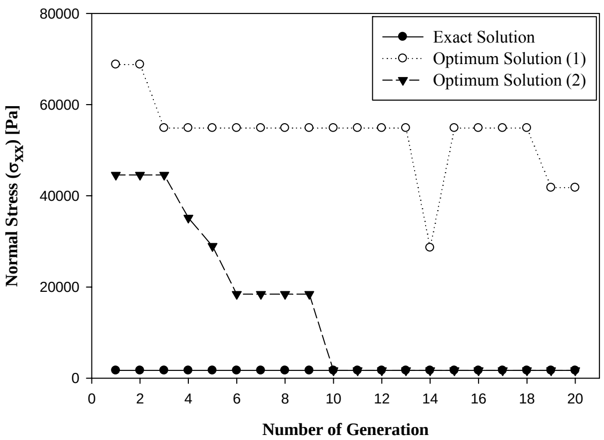

Fig. 2. The variation of normal stresses with respect to the number of generation for C – C boundary condition

In Figure 2, 3, 4, the variation of normal stresses with respect to the number of generation are presented for C – C, C – F and S – S boundary conditions. Although the minimum stress values are obtained for all boundary conditions, the ones obtained with failure criteria converge faster than the others as illustrated. The minimum values are obtained in 10th, 3rd and 2nd generation for C – C, C – F and S – S boundary conditions respectively.

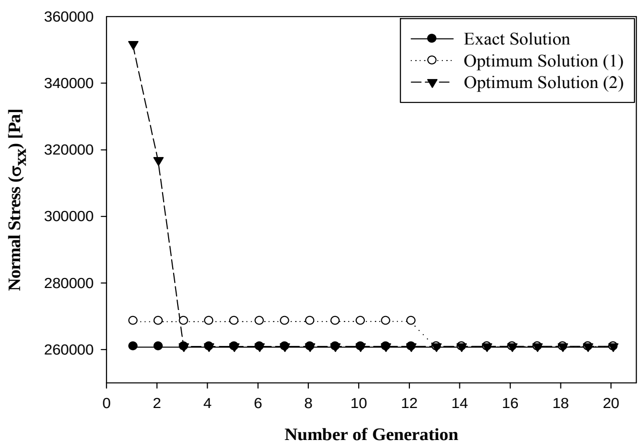

Fig. 3. The variation of normal stresses with respect to the number of generation for C – F boundary condition

In Figure 5, 6, 7, the variation of shear stresses with respect to the number of generation are presented for various boundary conditions. In common with the normal stresses, minimum shear stresses are generally obtained for both cases. On the other hand, stress parameters with failure criteria generally converge faster to the exact values as well as the previous figures for normal stresses.

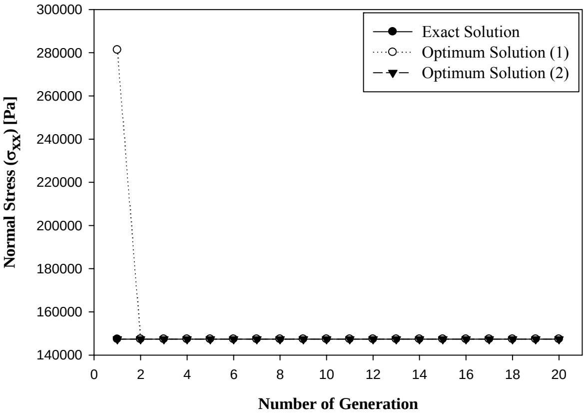

Fig. 4. The variation of normal stresses with respect to the number of generation for S – S boundary condition

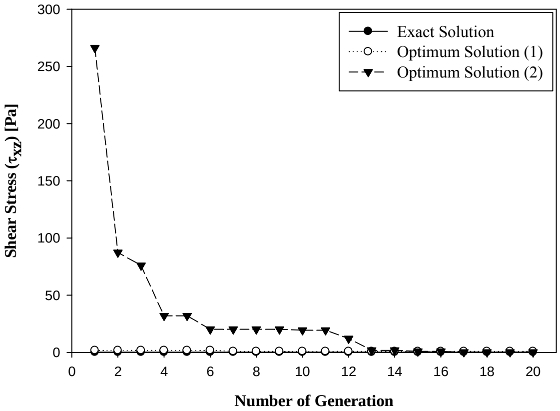

Fig. 5. The variation of shear stresses with respect to the number of generation for C – C boundary condition

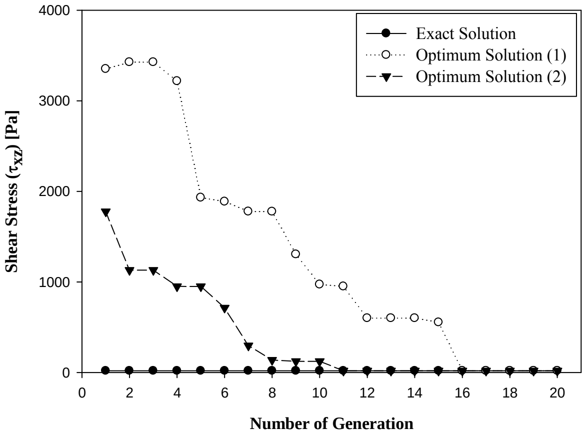

Fig. 6. The variation of shear stresses with respect to the number of generation for C – F boundary condition

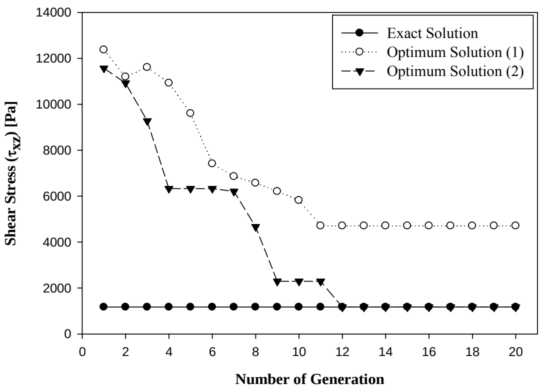

Fig. 7. The variation of shear stresses with respect to the number of generation for S – S boundary condition

The minimum values are obtained in 13th, 11th and 12th generation for C – C, C – F and S – S boundary conditions respectively. The convergence to the minimum can be increased by use of a bigger population and various crossover and mutation ratios.

CONCLUSION

In the study, the optimization of stacking sequences of a laminated composite beam is carried out for minimum bending stresses. Since it is very important for a structure to withstand the external and interior effects, the safety of the design is controlled by use of various failure criteria in the optimization process. The optimization problem is considered as a minimization problem and optimum values are investigated. For both of the bending stresses and various boundary conditions, minimum bending stresses are identical with the ones obtained by exact solution. The study can be extended by use of different beam theories, failure criteria, material properties and number of layers.

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