Р а з д е л 3

ЕЛЕКТРОТЕХНИКА, ЕЛЕКТРОНИКА И АВТОМАТИЗАЦИЯ

S e c t i o n 3

ELECTRICAL ENGINEERING, ELECTRONICS AND AUTOMATION

IMPEDANCE FREQUENCY RESPONSE MEASUREMENT WITH MATLAB IMPLEMENTATION

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Abstract

In the present paper a Matlab®-based program is developed for measuring the impedance frequency response of two-port circuits using data acquisition system NI USB-6211, Matlab® Data Acquisition Toolbox and Matlab® Signal Processing Toolbox. A brief theoretical analysis is given including the concept of complex impedance and frequency response, the Maximum Likelihood method for signal amplitude and phase estimation, and a simple method for phase correction due to non-instantaneous sampling in DAQ-system. Block diagrams of the experimental setup and the program algorithm are presented. A comparison and analysis between the theoretical and experimental data has been made and conclusions about the measurements are drawn.

The proposed Matlab® program can be downloaded for free, from the link given at the end of the paper. With slight alterations in the program code the latter could be used for frequency response measurements of four-port circuits.

Keywords: Impedance, Frequency response, Magnitude, Phase, Measurement, Matlab®.

INTRODUCTION

The complex impedance is an important parameter characterizing the electrical and electronic circuits and components. In the field of audio technique the impedance frequency response is of interest when examining electroacoustic transducers (loudspeakers, hydroacoustic antennas etc.). Being aware of the frequency response amplitude and phase of the load is important for the analysis and the design of amplifiers used by the transducers [1, 2].

There is a variety of methods for measuring the (complex) impedance: I-V method, bridge method, network-analysis method, etc [3]. Since one is interested in the complex impedance frequency response (i.e. the complex impedance values in some frequency range) the device under test (i.e. the two-port circuit) must be exited with test signals with all relevant frequencies [4].

The excitation could be performed via single-tone sine wave signal with step-by-step incrementing frequency, linear FM signal (chirp, sweep) or white/pink noise signal.

In the current paper the I-V method combined with frequency response measurement with a sine wave signal with stepwise increasing frequency and frequency domain

processing (via DFT) are used. Brief theoretical analyses of the method as well as its practical application by using data acquisition (DAQ) system NI USB-6211 are shown. Matlab® software program for measurement control and post-processing of the results is provided. Due to the generalized approach, the developed software can be also used for measuring the amplitude and phase frequency responses of four-port circuits.

THEORETICAL BACKGROUND

Impedance frequency response

The test sine wave signal at the outputs of the two-port circuit (Fig. 1) is voltage $u(t)$ of the type [5]:

$$x(t) = X_m \sin(2\pi f_0 t + \varphi_X) , \qquad (1)$$

where: X_m is a amplitude of the signal;

f_0 – frequency of the signal;

t – time;

φ_X – phase of the signal.

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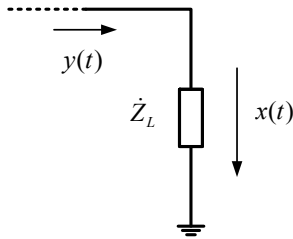[**] *Phone: 0886001599; e-mail: iliyan.j.iliev@gmail.com*

Fig. 1. Theory setup

The system reacts by a current flow $i(t)$:

$$y(t) = Y_m \sin(2\pi f_0 t + \varphi_Y) . \tag{2}$$

The voltage and the current in the circuit are bound by [6]:

$$\dot{Z}_L = \frac{\dot{X}}{\dot{Y}} = \left|\dot{Z}_L\right| e^{j\varphi_L} = \frac{X_m}{Y_m} e^{j(\varphi_X - \varphi_Y)} , \tag{3}$$

where: $\dot{Z}_L$ is a complex impedance;

φ_L – phase of the impedance.

By applying a sine wave signal whose frequency increases with time one can obtain information on the frequency dependency of the impedance's module and phase, i.e. the complex impedance can be found:

$$\dot{Z}_L(\omega) = Z_L(\omega) \angle \varphi_L(\omega) . \tag{4}$$

From the analysis of (3) one can deduce that the exact amplitude and phase of the voltage and the current in the circuit should be determined. The measurement is carried out by using a computer-based DAQ-system where the data at the system's output has a digital representation – samples, which suggests a digital processing of the signals.
Single tone amplitude and phase estimation
Different approaches for determining the parameters of sine wave signals are given in [7, 8, 9]. The most appropriate method when the ratio between signal and noise is a-priori high is the Maximum Likelihood (ML) Method [10].

If one applies DFT on the sampled and weighed input $x_w[n]$ and output reaction $y_w[n]$ (i.e. voltage and current) the corresponding spectrums are found [11]:

$$\dot{X}[k] = \frac{1}{N_x} \sum_{1}^{N_x} x_w[n] e^{-j\frac{2\pi kn}{N_x}} , \tag{5}$$

$$\dot{Y}[k] = \frac{1}{N_y} \sum_{1}^{N_y} y_w[n] e^{-j\frac{2\pi kn}{N_y}} , \tag{6}$$

where: N is a number of samples;

k – number of the spectral component.

Then the amplitude and the phase of the signals of interest are [10]:

$$X_m = \left|\dot{X}[k_{xm}]\right|, \qquad \varphi_X = \arg\left(\dot{X}[k_{xm}]\right), \tag{7}$$

$$Y_m = \left|\dot{Y}[k_{ym}]\right|, \qquad \varphi_Y = \arg\left(\dot{Y}[k_{ym}]\right), \tag{8}$$

where $\arg[\cdot]$ is the argument (or phase) of $[\cdot]$.

In Eqs. (7) and (8)

$$k_m = \arg \max_k \left|A[k]\right| \tag{9}$$

is the ML estimate of k – the value of k that maximizes the function $A(k)$ [10].

The signals are processed by means of a window function that provides an exact estimate of their amplitude – a Flat Top Window (FTW), whose parameters are given in [12, 13]. In this specific setup a fourth-order normalized FTW is used, with parameters:
- maximum side-lobe level (MSLL): -86,5 dB;
- scalloping loss (SL): 0,008;
- normalized equivalent noise bandwidth (NENBW): 3,771 bins;
- coherent amplification (CA): 0,215.

The width of NENBW is the main disadvantage of the FTW but it is compensated by the great number of acquired samples N (i.e. the small frequency bin width), so ENBW is narrow enough.

One must be aware that:
- the buffer amplifier's frequency response doesn't affect the measurement of the impedance frequency response;
- processing in the frequency domain instead in the time domain guarantees greater noise immunity of the measurement.

Phase correction due to non-simultaneous sampling
The measurement of the impedance phase φ_L could be seriously affected by the non-simultaneous sampling of the analog inputs. For multichannel DAQ-systems the channel-to-channel skew is defined as the difference Δt_s between the sampling times for each data channel. The channel skew Δt_s introduces a phase difference between two adjacent channels:

$$\Delta \psi = 2\pi \cdot \Delta t_s \cdot f_0 . \tag{10}$$

Therefore the correction of the φ_L, must be done accordingly:

$$\varphi_{Lcorr} = \varphi_L + \Delta \psi , \tag{11}$$

since the Δt_s appears in such a way so it seems the second channel ($y[n]$) leads the first one ($x[n]$), i.e. the voltage lags the current even with resistive load.
Transient suppression
Since the sine wave test signal has a finite duration and rectangular envelope, at the beginning and the end of the system response signal will be a turn-on and turn-off transients, respectively. One can minimize the transient by starting the system excitation at a point where the sine wave is zero and by allowing the transient to die out before collecting data [14]. For that reason a time-selective method – strobe is used [15]. Another approach is to change the envelope of the excitation signal, as is shown in [16].

EXPERIMENTAL SETUP

At present the measurements are performed with DAQ-systems [17], which provide automatization of the measurement process and easy post-processing of the obtained data.

In Fig. 2 an experimental setup block diagram is presented. A DAQ-system NI USB-6211 [18], a laptop and a Matlab®-based program are used. The test signal (1) from the analog output of DAQ-system is led to the two-port circuit input. In order to determine the impedance frequency response by (3), the voltage at the ends of the two-port circuit has been measured. So has been the current – by means of shunt resistor R_{sh}.
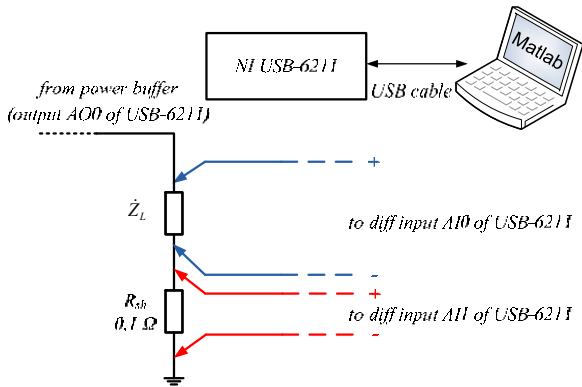

Fig. 2. Experimental setup block diagram

Three types of two-port circuits are used as an object of this study: a resistor, a serial *RC*-circuit and Dynaudio 17W-75 loudspeaker [19]. The parameters of the two-port circuits are given in Tab.1 and the equivalent circuit of the loudspeaker is shown in Fig. 3 [20].

Tab. 1. Parameters of the measured two-port circuits

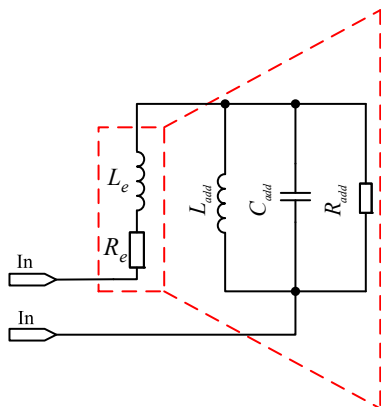| Configuration	Component values
R	$R = 10{,}1\ \Omega$
Serial RC	$R = 10{,}1\ \Omega,\ C = 30\ \mu F$
Loudspeaker 17W-75	$R_e = 5{,}1\ \Omega,\ L_e = 230\ \mu H,\ R_{add} = 13\ \Omega,$ $L_{add} = 0{,}018\ H,\ C_{add} = 858\ \mu F$

Fig. 3. Electrical equivalent circuit of the loudspeaker

The frequency response measurement technique is implemented in Matlab® as `freqrespmeasure` function, part of which is given below:

```
% signals length
xlen = length(x);
ylen = length(y);

% window preparation
xwin = flattopwin(xlen, 'periodic');
ywin = flattopwin(ylen, 'periodic');

% define the coherent amplification of the window
Kx = sum(xwin)/xlen;
Ky = sum(ywin)/ylen;

% fft of the first signal
X = fft(x.*xwin);

% fft of the second signal
Y = fft(y.*ywin);

% spectral peaks detection
[~, indx] = max(abs(X));
[~, indy] = max(abs(Y));
% freqeuncy response amplitude
Xamp = abs(X(indx))/xlen/Kx;
Yamp = abs(Y(indy))/ylen/Ky;
Amp = Yamp/Xamp;

% frequency response phase
Ph = angle(Y(indy)) - angle(X(indx));
```

In Fig. 4 a block diagram of the program algorithm used for measurement is presented.
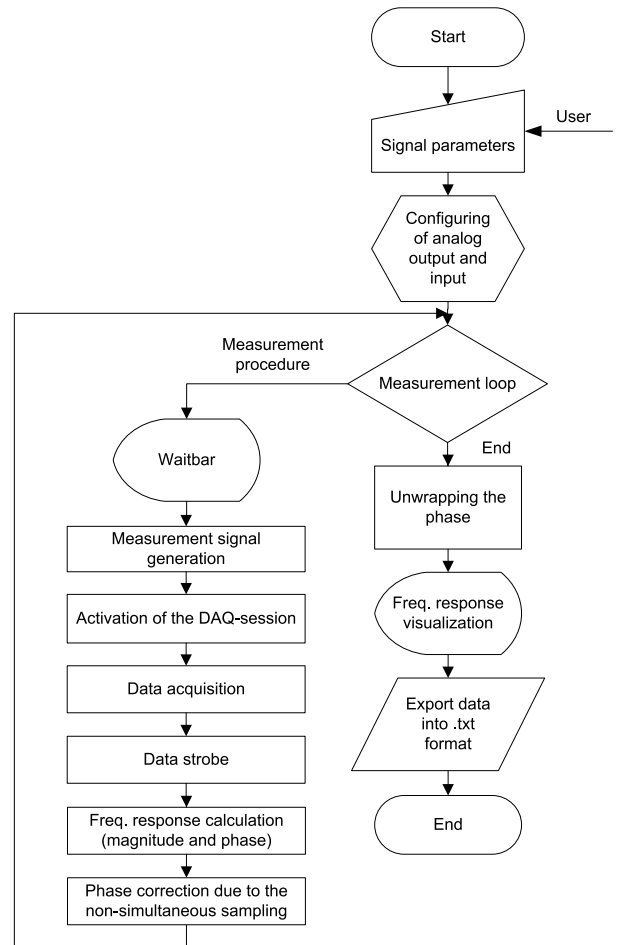

Fig. 4. Program algorithm

The main program starts with adjustment of the measurement settings:

```
% measurement settings
fsout = 250000;
fsin = 125000;
T = 0.5;
inrangech1 = [-10 10];
inrangech2 = [-0.2 0.2];
calout = 1;
calch1 = 1;
calch2 = 10;
```

An analog output and an analog input must be prepared:

```
% preparation of the analog output
ao = analogoutput('nidaq', 'Dev1');
addchannel(ao, 0);
set(ao, 'SampleRate', fsout)
set(ao, 'TriggerType', 'Immediate')
set(ao, 'ExternalTriggerDriveLine', 'PFI4')

% preparation of the analog input
ai = analoginput('nidaq', 'Dev1');
addchannel(ai, [0 1]);
set(ai, 'SampleRate', fsin)
set(ai, 'InputType', 'Differential')
set(ai.Channel(1), 'InputRange', inrangech1)
set(ai.Channel(2), 'InputRange', inrangech2)
set(ai, 'TriggerType', 'HwDigital')
set(ai, 'HwDigitalTriggerSource', 'PFI3')
set(ai, 'TriggerCondition', 'PositiveEdge')
set(ai, 'SamplesPerTrigger', round(T*fsin))
chskew = get(ai, 'ChannelSkew');
```

The following code is the measurement procedure:

```
% measurement procedure
for c = 1:freqveclen

    % generation of the signal
    f = Freqvec(1, c);
    outdata = sin(2*pi*f*t)'*calout;

    % activate the DAQ session
    putsample(ao, 0)
    putdata(ao, outdata)
    start([ai ao])

    % get data
    wait(ai, 1.1*T+0.5)
    indata = getdata(ai);

    % stop the DAQ session
    stop([ai ao])
    flushdata(ai)

    % strobe the data
    u = indata(0.1*end:0.9*end, 1)*calch1;
    i = indata(0.1*end:0.9*end, 2)*calch2;

    % calculate the impedance frequency response
    [ModZ, PhZ] = freqrespmeasure(i, u);

    % correction of the phase error due to non-
simultaneous sampling
    phi = 2*pi*f*chskew;
    PhZ = PhZ + phi;

    % save the data into frequency response
vectors
    ModZvec(1, c) = ModZ;
    PhZvec(1, c) = PhZ;

    % waitbar
    waitbar(c/freqveclen, h, ['Measurement at '
num2str(f) ' Hz'])

end
```

Finally, the measurement results are visualized:

```
% plot the impedance frequency response
figure(1)
subplot(2, 1, 1)
semilogx(Freqvec, ModZvec, 'r', 'LineWidth', 2)
grid on
xlim([min(Freqvec) max(Freqvec)])
ylim([0.8*min(ModZvec) 1.2*max(ModZvec)])
set(gca, 'FontName', 'Times New Roman',
'FontSize', 14)
set(gca, 'XTickLabel', num2str(get(gca,
'XTick').'))
xlabel('Frequency, Hz')
ylabel('|Z|, \Omega')
title('Impedance frequency response')
subplot(2, 1, 2)
semilogx(Freqvec, PhZvec, 'r', 'LineWidth', 2)
grid on
xlim([min(Freqvec) max(Freqvec)])
ylim([min(PhZvec)-10 max(PhZvec)+10])
set(gca, 'FontName', 'Times New Roman',
'FontSize', 14)
xlabel('Frequency, Hz')
ylabel('\phi, \circ')
```

In addition the obtained experimental data can be saved as .txt file for further processing:

```
% save the data in .txt file
D = [Freqvec' ModZvec' PhZvec'];
dlmwrite('dataLSP.txt', D, 'delimiter', ' ',
'newline', 'pc');
```

EXPERIMENTAL RESULTS

The results from the measurements of the two-port circuits are given in Fig. 5, 6 and 7. The result from the measurement of the non-inductive resistor R is used for assessing the accuracy of the method, as follows:

- amplitude accuracy: less than $\pm$ 0,2 %;

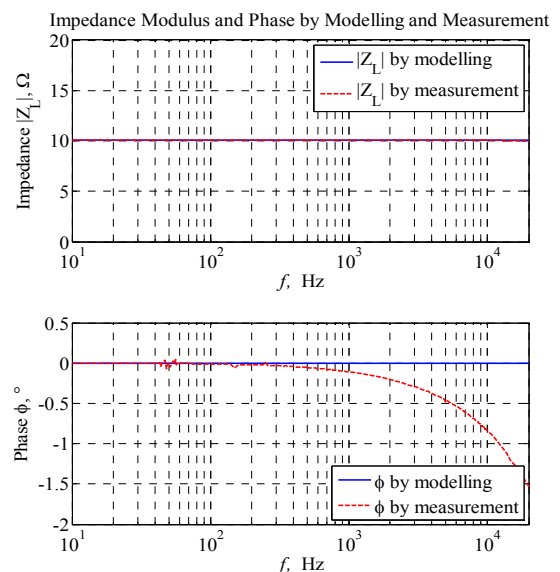- phase accuracy: less than $-1,5\,°$.

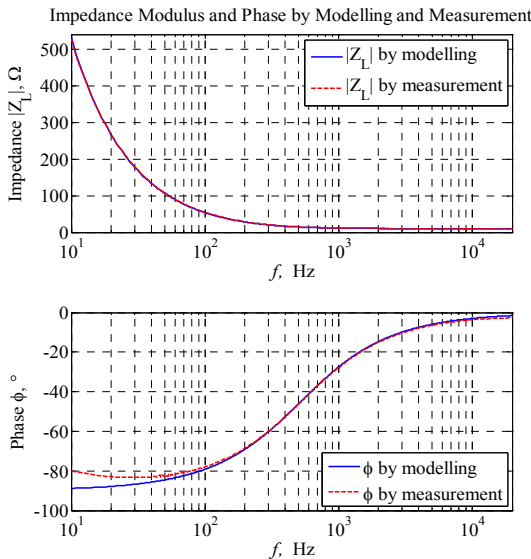*Fig. 5. Impedance modelling and measurement of a resistor R*

Fig. 6. Impedance modelling and measurement of a serial RC-circuit

The difference between the simulated and measured impedance response phase of the serial *RC*-circuit in lower frequency range is due to the significant losses in the capacitor.
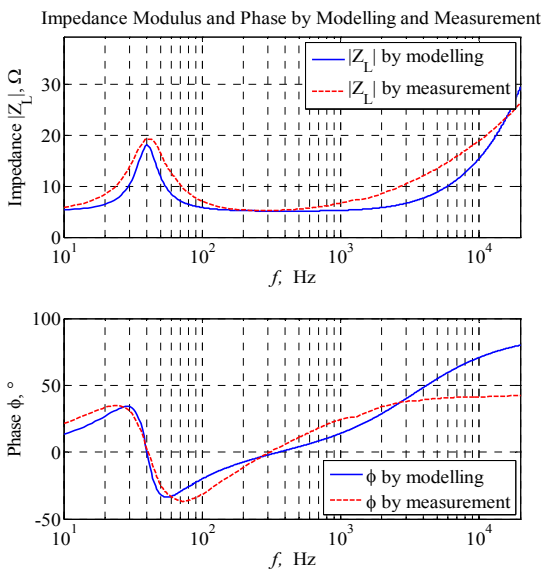

Fig. 7. Impedance frequency response modelling and measurement of a loudspeaker

Other differences between the simulated and experimental results from the loudspeaker's measurement can be explained with the simplified model used.

CONCLUSION

In the current paper a specific application of Matlab®-based software program for measuring the impedance frequency response of two-port circuit is proposed. It uses a sine wave signal with stepwise increasing frequency. By DFT and ML estimation of the amplitudes and the phases of the voltage across- and the current through- the two-port circuit are measured. Then the impedance modulus and phase is calculated by Eq. (3).

The results from the measurement validated the experimental setup and the author's (H. Zhivomirov) software program.

With slight alterations in the proposed Matlab® code the developed software can be successfully used for measuring four-port circuit frequency responses.

The presented program code is accessible at: http://www.mathworks.com/matlabcentral/fileexchange/50 637-impedance-frequency-response-measurement-with-matlab-implementation.

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