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PERFORMANCE EVALUATION OF LAMINAR FULLY DEVELOPED FLOW THROUGH TRAPEZOIDAL AND HEXAGONAL DUCTS SUBJECTED TO H1 BOUNDARY CONDITION. PART 2

Valentin M. Petkov   Ventsislav D. Zimparov*

Technical University of Gabrovo

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Abstract

Extended performance evaluation criteria (ExPEC) have been used to assess the performance characteristics of single-phase fully developed laminar flow through bundle of tubes with trapezoidal and hexagonal ducts. The bundle with circular tubes has been used as a reference heat transfer unit. The H1 boundary condition has been selected as thermal boundary condition. The performance characteristics of the heat unit with non-circular tubes have been evaluated and compared to those of the reference unit for different objectives and constraints imposed. As a common constraint, the hydraulic diameter of the non-circular duct has been specified. The results showed that only in the case VG-1 the benefit can be obtained for the values of the irreversibility ratio ϕ_o □ 1.

Keywords: performance evaluation criteria, single-phase laminar flow, trapezoidal and hexagonal ducts, entropy generation

Nomenclature

- A heat transfer surface area $[\mathrm{m^2}]$
- c_p specific heat capacity $[\mathrm{J\,kg^{-1}K^{-1}}]$
- D reference circular tube diameter [m]
- D_h hydraulic diameter [m]
- h heat transfer coefficient $[\mathrm{W\,m^{-2}\,K^{1}}]$
- k thermal conductivity $[\mathrm{W\,m^{-1}\,K^{-1}}]$
- L tube length [m]
- $\dot{m}$ mass flow rate in tube $[\mathrm{kg\,s^{-1}}]$
- N_t number of tubes
- P pumping power [W]
- p wetted perimeter [m]
- Δp pressure drop [Pa]
- $\dot{Q}$ heat transfer rate [W]
- $\dot{S}_{gen}$ entropy generation rate $[\mathrm{W\,K^{-1}}]$
- T temperature [K]
- ΔT wall-to-fluid temperature difference [K]
- V volume of tubes $[\mathrm{m^3}]$
- W mass flow rate in heat exchanger $[\mathrm{kg\,s^{-1}}]$
- x axial distance along the tube [m]

Greek symbols

- ϑ temperature difference, $T_w - T$
- μ dynamic viscosity $[\mathrm{Pa\,s}]$
- ρ fluid density $[\mathrm{kg\,m^{-3}}]$

Dimensionless groups

- A_* dimensionless heat transfer surface, $A_w / A_{w,c}$
- D_* dimensionless tube diameter, D_h / D_c
- L_* dimensionless tube length, L / L_c
- f Fanning friction factor
- f_* Fanning friction factor ratio, f / f_c
- Nu Nusselt number
- Nu_* Nusselt number ratio, Nu / Nu_c
- N_S augmentation entropy generation number
- N_* ratio of number of tubes, $N_t / N_{t,c}$
- NTU heat transfer units, $4St\,L/D$
- Pr Prandtl number
- P_* dimensionless pumping power, P / P_c
- Q_* dimensionless heat transfer rate, $\dot{Q}/\dot{Q}_c$
- Re Reynolds number
- Re_* Reynolds number ratio, $\mathrm{Re}/\mathrm{Re}_c$
- St Stanton number
- ΔT_i^* dimensionless inlet temperature difference, $\Delta T_i / \Delta T_{i,c}$
- V_* volume ratio, V / V_c
- W_* dimensionless mass flow rate, W / W_c
- χ shape factor, p / D_h
- χ_* ratio of shape factors, χ / χ_c
- ε_* ratio of heat exchanger effectiveness, $\varepsilon / \varepsilon_c$

*tel: (066 827) e-mail: vdzim@tugab.bg

τ dimensionless temperature difference, $\Delta T / T$
ϕ_o irreversibility distribution ratio

Subscripts
c circular tube
f fluid
i value at x = 0
m mean
o value at x = L
w wall

1. Introduction

There has been considerable work on laminar forced-convective heat transfer in non-circular ducts reported in the literature. Shah and London [1], and Shah and Bhatti [2] give extended reviews of a large number of these studies. In the more recent literature, several different flow cross-section geometries for newer compact heat exchanger applications have been studied. They include double-sine [3], circular segment [4], semi-circular [5] and several other unusual duct shapes.

Duct geometries as single- and double-trapezoidal (hexagonal) represent flow channels of a variety of compact heat exchangers. The double-trapezoidal duct shape is encountered in lamella type compact heat exchanger, which find extensive usage in chemical industry [6-8]. Plate heat exchangers are also used in a wide range of applications including food and chemical processing, refrigeration, and waste-heat recovery [9]. The single-trapezoidal channel is employed in plate-fin heat exchangers [8], and micro-channel electronic cooling modules [10].

Due to smaller system dimensions, the hydraulic diameter of flow channels in such heat exchangers are small and the length-to-diameter ratio, L / D_h is relatively large. Due to these length scales and the viscous nature of the fluids being handled, the flow is usually laminar with fully developed conditions. It is therefore important to investigate the performance characteristics of different ducted flows, particular in the laminar regime.

On the basis of the first-law analysis Webb [11] and Webb and Bergles [12] have proposed performance evaluation criteria (PEC) that define the performance benefits of an exchanger having augmented surfaces, relative to standard exchanger with smooth surfaces subject to various objectives and design constraints. A thermodynamic basis to evaluate the merit of augmentation techniques by second-law analysis has been proposed by Bejan [13,14] who developed the entropy generation minimization (EGM) method.

This method has been used as a general criterion for estimating and minimizing the irreversibilities and optimum-design method for heat exchangers. The coupling between fluid flow and heat transfer irreversibilities suggested that the geometry and operating conditions can be optimized to minimize the overall entropy generation. The method has been extended by Zimparov [15,16] including the effect of fluid temperature variation along the length of a tubular heat transfer unit, and new information has been added assessing two objectives simultaneously. The EGM method combined with the first law analysis provides the most powerful tool for the analysis of the thermal performance of any augmentation technique.

For any duct with non-circular shape, the size is determined by either the hydraulic diameter D_h, or the cross-sectional area A_f, since these parameters are related through the shape factor $\chi = 4A_f / D_h^2$. In this regard, two different common constraints can be imposed – specified cross-sectional area $A_f^* = 1$, or specified hydraulic diameter of the ducts, $D_* = 1$. Performance evaluation of laminar fully-developed flow in ducts with non-circular shapes subjected to H1 boundary condition and common constraint $A_f^* = 1$ have been recently presented in [17,18].

The rationale of the present study is to evaluate the thermal performance of laminar fully-developed flow in a bundle with trapezoidal and hexagonal ducts. Figure 1 presents the geometrical details of trapezoidal and double trapezoidal (hexagonal) duct [19]. The boundary condition is H1 (constant wall heat flux) with a common constraint, $D_* = 1$. In this case, the cross- sectional area of the duct is a consequence, $A_f^* = \chi_*$. The bundle of circular tubes has been used as a reference heat transfer unit. Using the first and second laws simultaneously, the performance characteristics of units with non-circular ducts have been evaluated for different objectives and constraints imposed and compared to those of the reference unit with circular tubes.

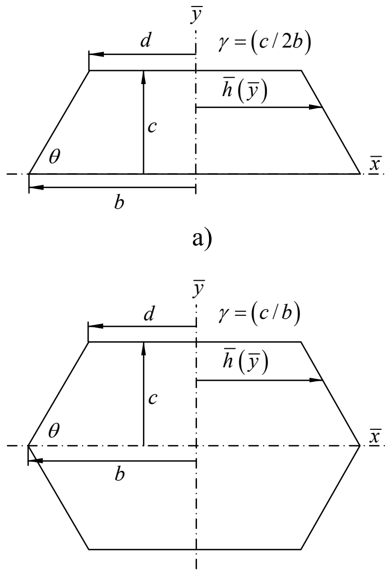

Fig. 1. Coordinate system and geometrical details of: a) trapezoidal duct; b) hexagonal duct

The model developed by Zimparov [16] on the entropy production theorem and the PEC as suggested by Webb [11] and Webb and Bergles [12] have been presented in details in Part 1 of this study [20].

2. Performance evaluation criteria

These criteria are based on the use of first and second law analyses in the pursuit of two objectives simultaneously. In this study the geometrical and regime parameters of the reference channel (smooth circular tube) are selected to fulfill the requirement of $4L_c / (D_c \mathrm{Re}\,\mathrm{Pr}) = 1$, corresponding to the fully-developed laminar flow in tube.

The values of the shape factor χ, friction factor f and Nusselt number Nu of trapezoidal and hexagonal ducts are taken and calculated from Sadasivam et al. [19]. While obtaining the augmentation entropy generation number, the irreversibility distribution ratio for the circular configuration, ϕ_o, varies in the range $10^{-3} \le \phi_o \le 10^3$.

In Part 1 of this study [20] we considered the implementation of criteria FG-1a, FG-2a, and VG-2a, where the first objective was the increased heat flow, $Q_* > 1$. In Part 2, the first objective is the decreased heat transfer area, $A_* < 1$, that is a goal of criteria FN-1 and VG-1.

2.1. Fixed number of tubes criteria (FN)

These criteria maintain constant number of tubes. The objective of FN-1 is reduced surface area, via reduced tubing length, for constant pumping power. Reduced flow rate will probably be required to satisfy the constant pumping power criterion. The objective of FN-2 is reduced pumping power.

2.1.1. Case FN-1

The objective functions of the case FN-1 are reduced heat transfer area $A_* < 1$ ($L_* < 1$), decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S A_* < 1$. The constraints imposed are: $N_* = 1$, $Q_* = 1$, $P_* = 1$, $\Delta T_i^* = 1$, and $D_* = 1$. The consequences of the constraints are $A_f^* = \chi_*$, $V_* = A_*$. The Eqs. (2-7) in Ref. [20] yield

$$W_* = \chi_* \mathrm{Re}_* = \varepsilon_*^{-1}, \tag{1}$$

$$L_* = \frac{\chi_* \varepsilon_*^2}{(f\,\mathrm{Re})_*}, \tag{2}$$

$$A_* = N_* \chi_* L_* D_* = \frac{\chi_*^2 \varepsilon_*^2}{(f\,\mathrm{Re})_*}. \tag{3}$$

where ε_* is calculated by Eq. (10), Ref. [20] $N_{S,P} = P_* = 1$,

$$\varepsilon_* = 1.229 \frac{Nu}{1+Nu}. \tag{4}$$

The Eqs. (3a-3c), Ref. [20] yield

$$T_o^* = 0.981 + 0.019\varepsilon_*, \tag{5}$$

$$N_{S,T} = \frac{(f\,\mathrm{Re})_*}{Nu_* T_o^* \chi_*^2 \varepsilon_*^2}, \tag{6}$$

and the augmentation entropy generation number N_S becomes

$$N_S = \frac{1}{1+\phi_o}\left(\frac{(f\,\mathrm{Re})_*}{Nu_* T_o^* \varepsilon_*^2 \chi_*^2} + \phi_o\right). \tag{7}$$

The calculated values of A_* by Eq. (3) are shown in Fig. 1. The variations of A_* with χ_* clearly show that the use of non-circular ducts as trapezoidal or hexagonal in the bundle cannot diminish the needed heat transfer area, and the first objective cannot be achieved. The variation of the angle θ does not have any effect on A_* either.

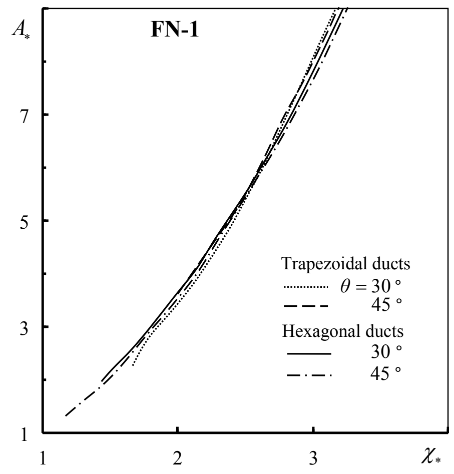

Fig. 1a. The variation of A_* with χ_* and θ

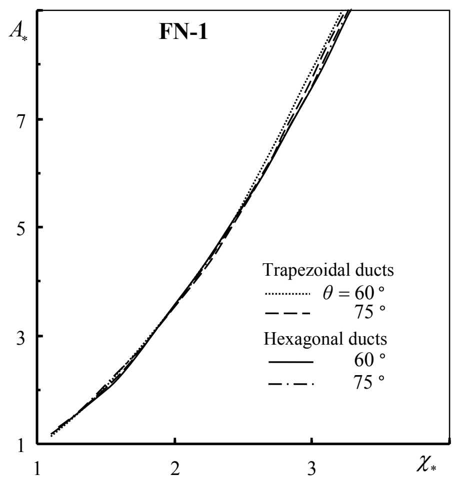

Fig. 1b. The variation of A_* with χ_* and θ

The implementation of the general criterion $N_S A_*$ also confirms the conclusion that no benefits should be expected since the second objective, $N_S A_* < 1$, cannot be reached either. The similarity of the images reveals that the variation of the angle does not have any effect on the performance. Fig. 2d shows that only for $\phi_o \ll 1$ ($\phi_o = 10^{-3}$) the second objective $N_S A_* < 1$ can be achieved. This is due to the small amount of the entropy generation because of the considerably.decrease of the duct flow rate. To be able to note any benefit, however, it is needed the two objectives to be achieved simultaneously.

Consequently, no benefits can be obtained through the replacement of traditional circular pipes in the bundle with ducts with no circular shape as trapezoidal or hexagonal.

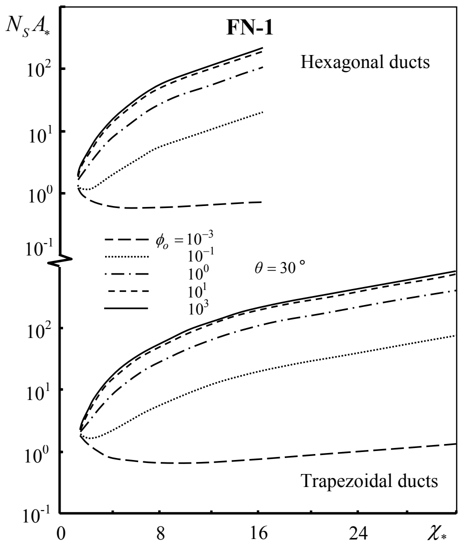

Fig. 2a. The variation of $N_S A_*$ with χ_* and θ

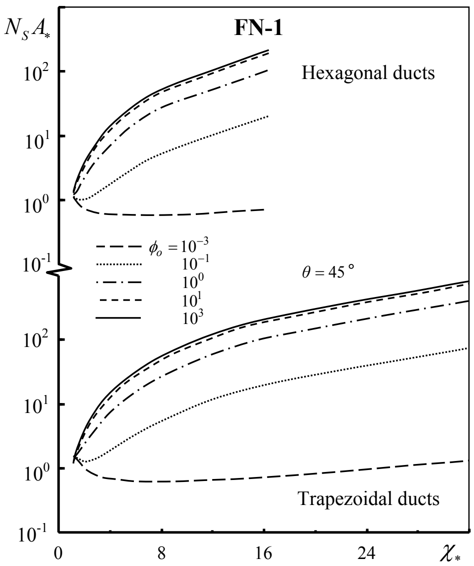

Fig. 2b. The variation of $N_S A_*$ with χ_* and θ

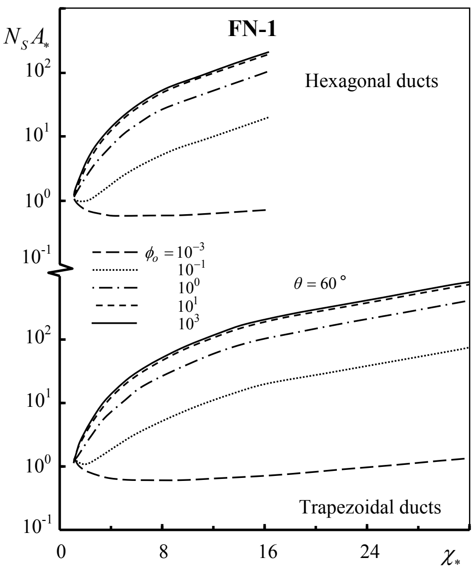

Fig. 2c. The variation of $N_S A_*$ with χ_* and θ

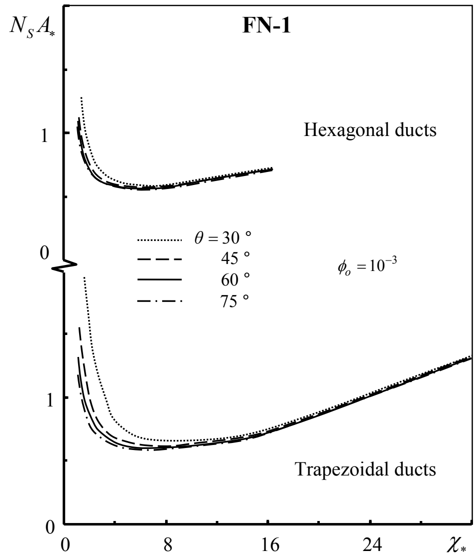

Fig. 2d. The variation of $N_S A_*$ with χ_* and θ

3.2. Variable geometry criteria (VG)

The criteria VG are applicable when the heat exchanger is “sized” for a required thermal duty with specified flow rate.

3.2.1 Case VG-1

The objective functions of the case VG-1 are lower heat transfer area $A_* < 1$, decreased entropy generation number $N_S < 1$, and simultaneous effect of the both of them $N_S A_* < 1$. The constraints imposed are: $Q_* = 1$, $W_* = 1$, $P_* = 1$, $\Delta T_i^* = 1$, $D_* = 1$. The consequences of the constraints are $L_* < 1$, $A_{f,tot}^* > 1$, $V_* < 1$. The Eqs. (2-7), Ref. [20], yield: $\varepsilon_* = 1$, $T_o^* = 1$, $N_{S,P} = P_* = 1$,

$$A_* = Nu_*^{-1}, \tag{8}$$

(since $Q_* = 1$, and $\Delta T_m^* = 1$),

$$N_* = \left[\frac{(f\,\mathrm{Re})_*}{\chi_*^2 Nu_*} \right]^{1/3}, \tag{9}$$

$$L_* = \left[\chi_* (f\,\mathrm{Re})_* Nu_*^2 \right]^{-1/3}, \tag{10}$$

$$\mathrm{Re}_* = \left[\frac{Nu_*}{\chi_* (f\,\mathrm{Re})_*} \right]^{1/3}, \tag{11}$$

$$N_{S,T} = \left[\frac{\chi_*^2 Nu_*}{(f\,\mathrm{Re})_*} \right]^{1/3}. \tag{12}$$

The augmentation entropy generation number N_S becomes

$$N_S = \frac{1}{1+\phi_o} \left\{ \frac{\chi_*^{2/3} Nu_*^{1/3}}{(f\,\mathrm{Re})_*^{1/3}} + \phi_o \right\}. \tag{13}$$

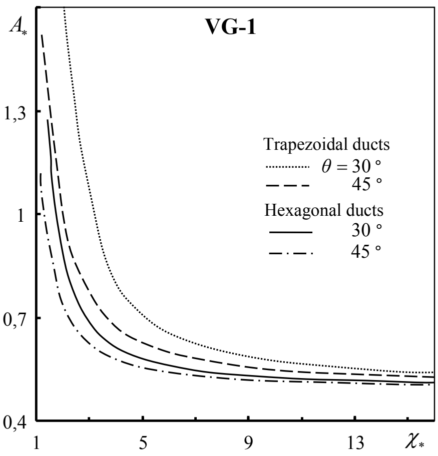

Fig. 3a. The variation of A_* with χ_* and θ

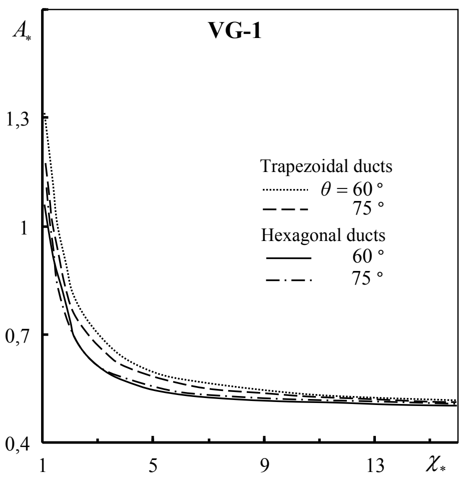

Fig. 3b. The variation of A_* with χ_* and θ

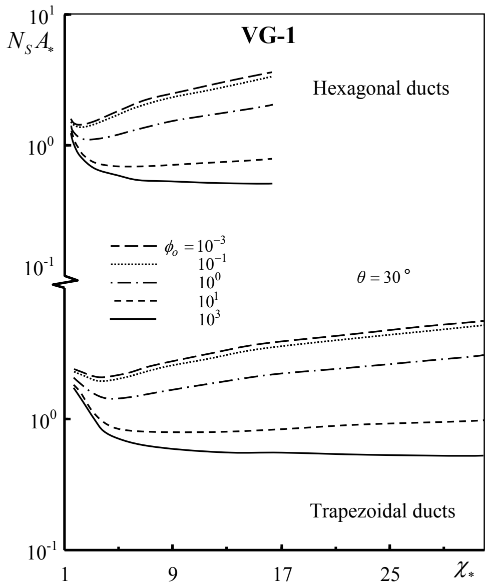

Fig. 4a. The variation of $N_S A_*$ with χ_* for trapezoidal and hexagonal ducts $(\theta = 30^{\circ})$

The calculated values of A_* by Eq. (8) are presented in Fig. 3 as the variation of A_* with χ_*. As seen, the first objective $A_* < 1$ can be achieved for both trapezoidal and hexagonal ducts if $\chi_* > 2.5$ and the benefit slightly increases with the increase of θ. The benefit is considerable and reaches almost 50% reduction of heat transfer surface for $\chi_* > 9$.

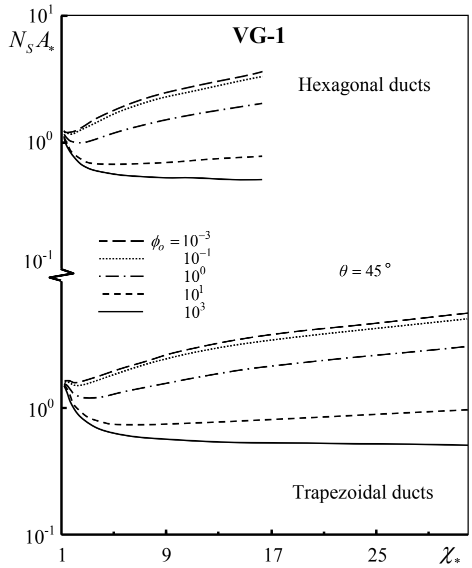

Fig. 4b. The variation of $N_S A_*$ with χ_* for trapezoidal and hexagonal ducts $(\theta = 45^{\circ})$

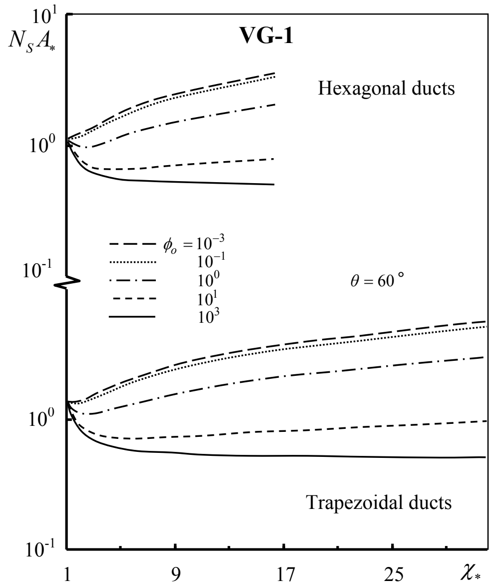

Fig. 4c. The variation of $N_S A_*$ with χ_* for trapezoidal and hexagonal ducts $(\theta = 75^{\circ})$

The use of the general criterion $N_S A_*$ also confirms the conclusion that benefit can be obtained since the second objective, $N_S A_* < 1$, can also be achieved, Fig. 4. The similarity of the images reveals that the variation of the angle θ have also small effect on the performance. The impact of ϕ_0, however, is significant. As seen, Fig. 4d, the second requirement, $N_S A_* < 1$, can be fulfilled only if $\phi_0 \geq 10$. It means, that the entropy generation due to the hydraulic resistances is dominated compared to the entropy generated by the heat transfer through the final temperature difference.

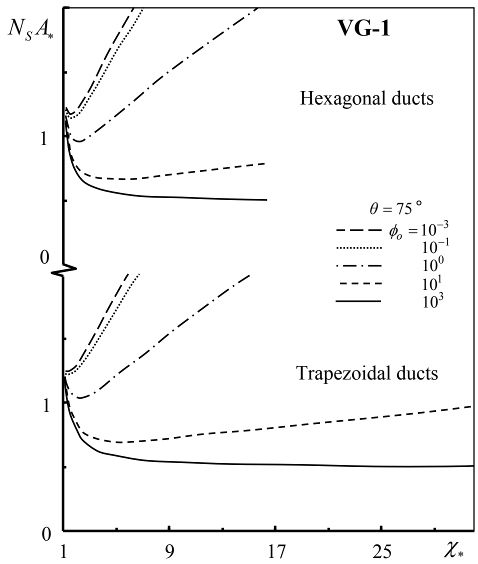

Fig. 4d. The variation of $N_S A_*$ with χ_* for trapezoidal and hexagonal ducts $(\theta = 75^{\circ})$

4. Conclusions

ExPEC have been used to assess the performance characteristics of single-phase fully developed laminar flow through bundle of tubes with trapezoidal and hexagonal ducts compared to bundle with standard circular pipes. The H1 boundary condition has been selected as thermal boundary condition with additional constraint of fixed hydraulic diameter, $D_* = 1$. The performance characteristics have been evaluated for the cases FN-1 and VG-1, where the first objective is reduction of heat transfer area, $A_* < 1$

The results showed that benefit can be obtained only in the case VG-1 for values of the irreversibility ratio $\phi_o > 10$, and it could be significant.

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Authors' information

Technical University of Gabrovo,
4 Hadji Dimitar, 5300 Gabrovo, Bulgaria.
Division of Fluid Power and Heat Transfer Engineering

Valentin Petkov is an Assistant Professor of thermodynamics and heat transfer at Gabrovo Technical University, Bulgaria. He received his BSc and PhD degrees from Gabrovo Technical University. His main research interests are enhanced heat transfer, compact heat exchangers, and thermodynamic optimization.

Ventsislav Zimparov is a Professor of thermodynamics and heat and mass transfer in Gabrovo Technical University, Bulgaria. He received his PhD in 1990 from Technical University of Sofia, Bulgaria and DSc degree in 2004. His main research interest are enhanced heat transfer, compact heat exchangers, heat exchanger networks, constructal theory and thermodynamic optimization. He has published more than 80 articles in well-recognized journals and proceedings.